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LETTERS TO THE EDITOR

COMMENTS ON "FUNDAMENTAL FREQUENCIES OF TIMOSHENKO BEAMS MOUNTED ON PASTERNAK FOUNDATION"

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In a recent and comprehensive Letter to the Editor [1] approximate explicit formulae have been derived by El-Mously for the fundamental natural frequency for vibration of Timoshenko beams mounted on Pasternak foundation.

It is the purpose of this note to mention other previous works in which the application of the Pasternak foundation model is the important subject of an engineering field of great interest.

Probably because of the difficulties in the estimation of the value of the soil parameters, as pointed out by Franciosi and Masi [2], and because of the complexities of the model, closed-form solutions are available only for the simplest cases and the free vibration frequencies must be numerically calculated even for simply supported beams [3]. Therefore, due to the importance of this class of studies, it is very useful to obtain data from different experimental tests in order to improve and validate theoretical models.

Wang and Stephens [4] showed the effect of rotatory inertia and shear deformation on the natural frequencies of a beam for various boundary conditions. This work was extended by Maurizi and Rosales [5] to rotating elastically restrained ends. Wang and Gagnon [6] completed the investigation of reference [4] by presenting the vibration of continuous Timoshenko beams.

In 1988, Filipich and Rosales [7, 8] determined the fundamental frequencies of Timoshenko beams resting in a Winkler–Pasternak (W–P) medium. They proposed the use of the variant of Rayleigh's method which allows an optimization of the approximate modal functions through a non-integer exponential parameter, originally suggested by Lord Rayleigh [9] and sufficiently employed in various eigenvalues and field problems. In the two papers mentioned the practical application of this technique yields very good results for hinged–hinged and clamped–clamped beams of uniform and variable cross section.

Subsequently, Yokoyama [10] developed a finite element procedure analyzing the flexural vibration of a uniform Timoshenko beam-column on a two-parameter elastic foundation. The governing matrix equation for small amplitude is derived from Hamilton's principle and several numerical examples are provided to show the effects of the different parameters on the natural frequencies of the beam-column. Additionally, the dynamic analysis of Timoshenko beams on two-parameter elastic foundations has been investigated by De Rosa [11] for two different foundation models of the second foundation parameter. As an extension of the approach, the free vibration frequencies were examined.

Finally, the Matsunaga's paper [12], recently published, offers approximate theories to predict the natural frequencies and buckling stress of deep beam-columns on two-parameter elastic foundations.

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AUTHOR'S REPLY

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The author wishes to thank Prof. Maurizi for his interest in the author's article [1], and would like to clarify the following. The main aim of the author's article was to derive "explicit" formulae for the fundamental natural frequencies for the vibration of finite Timoshenko-beams mounted on either finite or continuous Pasternak foundation. The article focuses on prismatic and initially straight beams, with six different sets of end restraints. The frequency equations of this soil–structure system are highly transcendental, and no attempt has been made to solve them, even in the simplest cases. The work presented